



An Analysis of Self Convolution windows using Matlab

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ABSTRACT

Windows function plays very important roles in Signal Processing. Sidelobes power in Windows affects the performance in communication systems. Self Convolution windows (SCW) obtain by convolution of the window function with itself leads to better sidelobe Side ratio (SLFOR). Various self convolution windows like (Triangular SCW) TSCW, (Hanning SCW) HSCW, Blackmanharris SCW) B-HSCW, (Cosine SCW) CSCW and (Nuttall SCW) NSCW are analysed using MATLAB software and the relative comparison is studied. Various self convolutional windows as discussed in the research paper could be used for beam forming, to reduce Out band and or to reduce sidelobe.

Key words: Window, Self convolution window, Sidelobes attenuation

1. INTRODUCTION

In Signal Processing and Communication Windows function [1]. has vital role, FIR Filter Design, Harmonic analysis and Spectral leakage reduction are few areas where windows function has significance importance. Main lobe width and the sidelobe are the two important parameters of any window function. Lesser mainlobe width and more sidelobe attenuation of a window are the two contradictory requirements of any communication system design. Increasing Sidelobe attenuation will increase the main lobe width and hence reducing the frequency resolution and vice versa. Self Convolution windows has been discussed in the literature [1-2]. to improve the sidelobe performance which reduces the leakage errors and the harmonic interference, which is one of the most important factor affecting Digital Filter performance. In [3]. there is a discussion on harmonic analysis of various window function using discrete fourier transform. In [4]. there is a discussion on new window function using time convolution and their spectral analysis. Organization of the research paper is follows in the manner as follows. In section 2 Frequency response & Time response of TSCW is studied . In section 3 Time and Frequency response of HSCW is discussed. In section 4 Frequency response & Time Response of B-HSCW is discussed. In section 5 Frequency response & Time Response of CSCW is done. In section 6 Frequency response & Time Response of NSCW is done In section 7 relative comparison is made between various self convolution windows.

2. TSCW

The Time domain and frequency domain expression of TSCW is given in [5]

$$W_T(\omega) = \frac{2}{M} \frac{e^{-jM\omega T/2} \left(\frac{\sin M\omega_s}{4}\right)^2}{\frac{\sin M\omega_s}{2}}$$

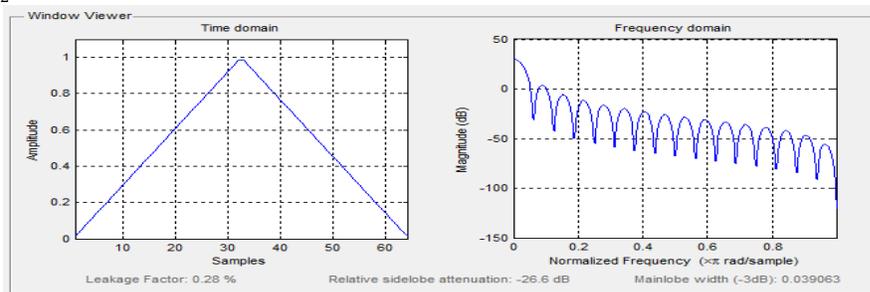


Fig 1. characteristics of Triangular window

p^{th} order TSCW is obtained as in equation(6)

$$W_p = w_1 * w_2 * w_3 \dots - p \text{ times}$$

(i).SPECTRAL CHARACTERISTICS OF TSCW

DFT of the p^{th} order TSCW is given in [5].

$$W_T(\omega_s) = \frac{2^p e^{-jk\pi}}{M} \left(\frac{\sin(\frac{\pi k}{2p})}{\sin(\pi k/N)}\right)^{2p}$$

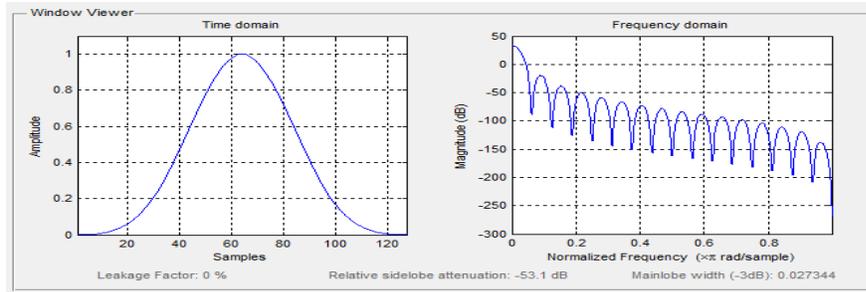


Fig 2. Characteristics of 2nd order TSCW

(ii). HSCW

The Time domain expression of HSCW is given in[6].

$$w_H(m) = .5 \left[1 - \cos\left(\frac{2\pi m}{M}\right) \right], m = 0, 1, \dots, M-1$$

The p^{th} order HSCW is obtained as in equation (9)

$$w_H^p(n) = w_H(m) * w_H(m) \dots p \text{ times}$$

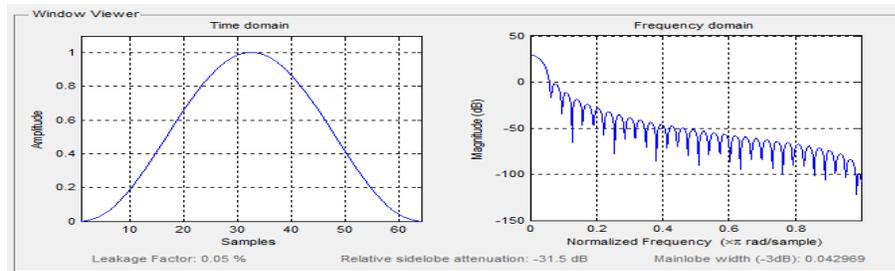


Fig 3. Characteristics of Hanning window

(iii).SPECTRAL CHARACTERISTICS OF HSCW

p^{th} order HSCW DFT is given in [6].

$$W_H^p(k) = \left\{ .5 W_R \left(\frac{2k\pi}{N} \right) + \left[.25 W_R \left(\frac{2k\pi}{N} - \frac{2\pi}{M} \right) + W_R \left(\frac{2k\pi}{N} + \frac{2\pi}{M} \right) \right] \right\}$$

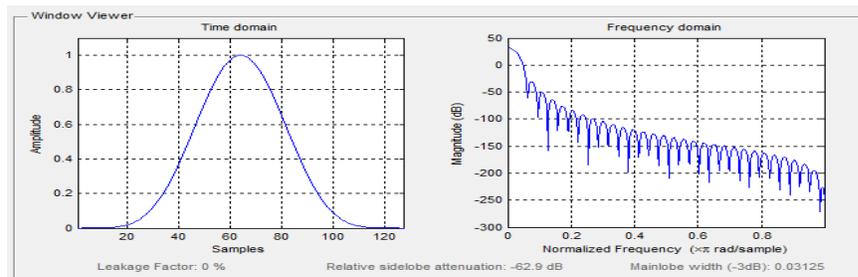


Fig 4. Characteristics of 2nd order HSCW

(iv). B-HSCW

The Time domain expression of BSCW is given in [7].

$$w(n) = a_0 - a_1 \cos\left(\frac{2\pi n}{M}\right) + a_2 \cos\left(\frac{4\pi n}{M}\right) - a_3 \cos\left(\frac{6\pi n}{M}\right), \quad n = 0, 1, \dots, M-1$$

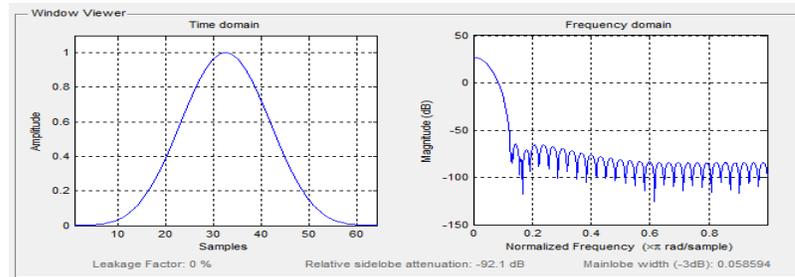


Fig 5. characteristics of 4 term B-HSCW

(v).SPECTRAL CHARACTERISTICS OF B-HSCW

The DFT of the p^{th} order BSCW is given in [7].

$$W_{B-H}^p(w) = \left\{ \sum_{n=0}^{M-1} a_n \left[W_R\left(\frac{2\pi k}{N} - \frac{2\pi n}{M}\right) + W_R\left(\frac{2\pi k}{N} + \frac{2\pi n}{M}\right) \right] \right\}^p$$

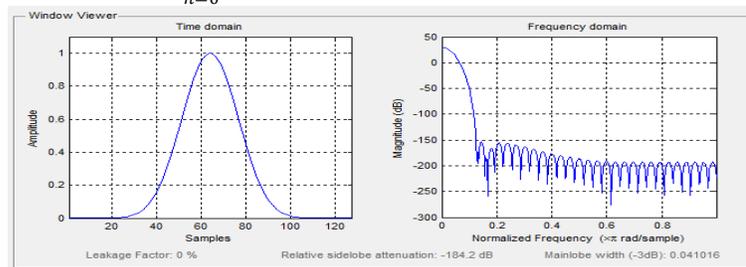


Fig 6. Characteristics of 2nd order 4 term B-HSCW

(vi).CSCW

CSCW expression in time domain is given in [8].

$$w(n) = \sum_{m=0}^{M-1} (-1)^m b_m \cos\left(\frac{2\pi mn}{M}\right), \quad n = 0, 1, \dots, M-1$$

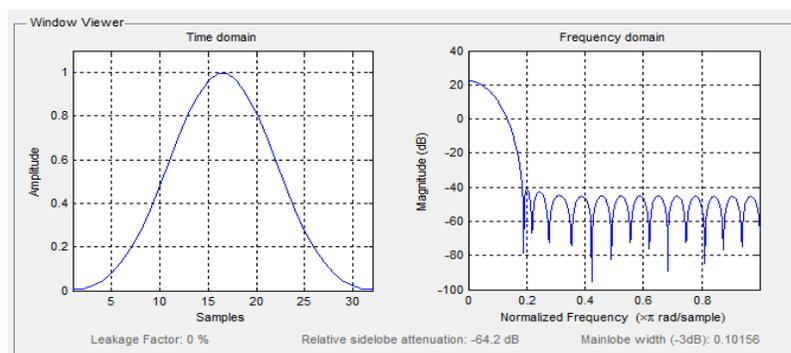


Fig 7. Characteristics of 3 term Cosine window

The P_{th} order CSCW is obtained by
 $w_c(n) = w(n) * w(n) \dots \dots \dots p \text{ times}$

(vii). SPECTRAL CHARACTERISTICS OF CSCW

P_{th} order CSCW DFT is given in [8].

$$W(L) = \sum_{m=0}^{M-1} (-1)^m \frac{b^m}{2} [W(L-m) + W(L-m)], n = 0, 1, 2 \dots \dots \dots N-1 \quad (15)$$

W_R , is the rectangular window DFT.

$$W_R(L) = \frac{\sin(L\pi)}{\sin(L\pi/N)} e^{-j(N-1)L\pi/N}$$

Convolution in time domain is equal to multiplication in frequency domain that leads to the DFT of CSCW is as given $W_c(L) = [W(L)]^p$

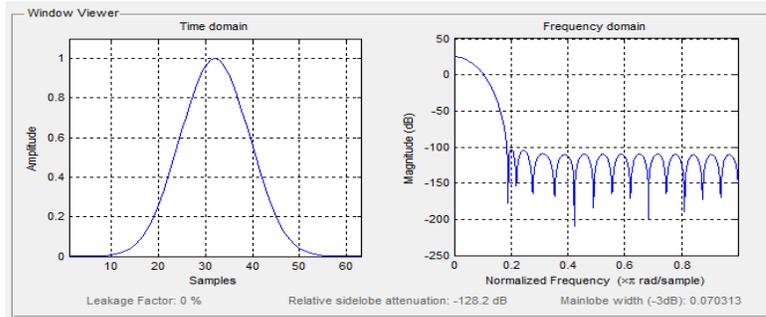


Fig 8. Characteristics of 3 term 2nd order CSCW

(viii). NSCW

The Time domain expression of NSCW is given in [9].

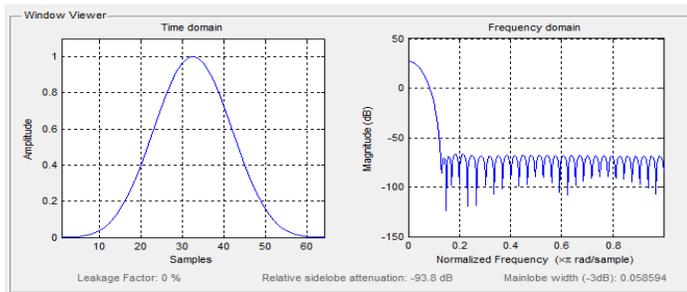


Fig 9: Characteristics of Nuttall Window

(ix). SPECTRAL CHARACTERISTICS OF NSCW

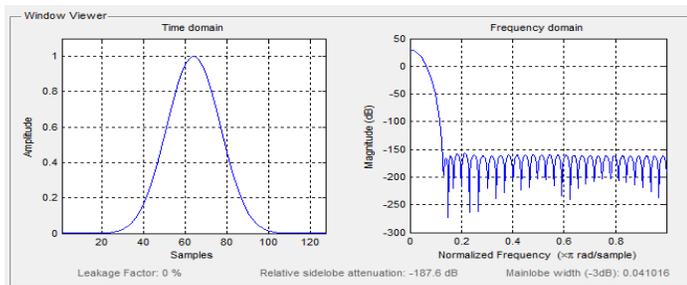


Fig 10: Characteristics of 2nd order NSCW

2.RESULT

The sidelobe attenuation and mainlobe width of various self convolution window (SCW) is presented in the Table (1)

Self Convolution Window	Sidelobe Attenuation	Main Lobe Width
TSCW	-53.1 db	0.02734
HSCW	-62.9 db	0.03125
BHSCW	-184.2 db	0.04106
CSCW	-128.2 db	0.070313
NSCW	-187.6 db	0.041016

Table 1. Comparison of various Self Convolution windows (SCW)

3. CONCLUSION

From Table (1) NSCW is having the maximum sidelobe attenuation, moreover for the same mainlobe width 2nd order NSCW is showing better sidelobe attenuation then 2nd order 3term B-HSCW. Of all the SCW discussed in the research paper KSCW is having the minimum mainlobe width.

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